Inverse analysis of continuous casting processes

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Abstract *This paper discusses an algorithm for phase change front identification in continuous casting. The problem is formulated as an inverse geometry problem, and the solution procedure utilizes temperature measurements inside the solid phase and sensitivity coefficients. The proposed algorithms make use of the boundary element method, with cubic boundary elements and Bezier splines employed for modelling the interface between the solid and liquid phases. A case study of continuous casting of copper is solved to demonstrate the main features of the proposed algorithms.*

1. Introduction

The continuous casting process of metals and alloys is a common procedure in the metallurgical industry. Typically, the liquid material flows into the mould (crystallizer), where the walls are cooled by flowing water. The solidifying ingot is then pulled by withdrawal rolls. The side surface of the ingot, below the mould, is very intensively cooled by water flowing out of the mould and sprayed over the surface, outside the crystallizer.

An accurate determination of the interface location between the liquid and solid phases is very important for the quality of the casting material. The estimation of this phase change front location can be found by using *direct modelling techniques* (Crank, 1984) such as the enthalpy method or front tracking algorithms or, as shown in this paper, by solving an *inverse geometry problem*.

Several previous works have dealt with inverse geometry problems (Bénard and Afshari, 1992; Kang and Zabaras, 1995; Nowak *et al.*, 2000; Tanaka *et al.*, 2000; Zabaras, 1990; Zabaras and Ruan, 1989). In particular, Zabaras and Ruan

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(1989) developed a formulation based on a deforming finite element method (FEM) and sensitivity coefficients to analyze one-dimensional inverse Stefan problems. Their formulation was applied to study the problem of calculating the position and velocity of the moving interface from the temperature measurements of two or more sensors (thermocouples) located inside the solid phase. Zabaras (1990) extended the deforming FEM formulation to two other problems: the first calculated the boundary heat flux history that would achieve a specified velocity and flux at the freezing front, while the second calculated the boundary heat flux and freezing front position, given the appropriate estimates of the temperature field in a specified number of sensors. Bénard and Afshari (1992) developed a sequential algorithm for the identification of the interface location, for one- and two-dimensional problems, using discrete measurements of temperature and heat flux at the fixed part of the solid boundary. Kang and Zabaras (1995) calculated the optimum history of boundary cooling conditions that resulted in a desired history of the freezing interface location and motion, for a two-dimensional conduction-driven solidification process.

In the present work following Nowak *et al.* (2000) and Tanaka *et al.* (2000), the solution procedure involves the application of the boundary element method (BEM) (Brebbia *et al.*, 1984; Wrobel and Aliabadi, 2002) to estimate the location of the phase change front, making use of temperature measurements inside the solid phase. This front is approximated by Bezier splines, and this is significant for the reduction of the number of design variables and, as a consequence, of the number of required measurements.

Identification of the position of the phase change front requires to build up a series of direct solutions, which gradually approach the correct location. Generally, inverse problems are ill-posed. Thus, there is a problem with the stability and uniqueness of solution (Goldman, 1997). In this paper, it is proposed that the iteration process (necessary because of the non-linear nature of the problem) is preceded by a lumping process. This allows the definition of an initial front position which guarantees convergence of the solution.

The measurements can be obtained by immersing thermocouples into the melt and allowing them to travel with the solidified material, until they are damaged. From certain relationships between time and location of nodes in the continuous casting process, even a limited number of thermocouples can provide a substantial amount of useful information. Alternatively, it is also possible to obtain temperature measurements by using an infrared camera. Although generally more accurate, temperatures have to be measured at the body surface outside the crystallizer, thus at some distance from the phase change front.

It is worth to stress that although temperature measurements in this work are limited only to the solid phase, they carry information on the heat transfer phenomena occurring on the solid-liquid interface. Moreover, mathematical models available for solids (based on heat conduction) are much more reliable than those for liquids where heat convection generally plays an important role.

2. Problem formulation

This section starts with a brief description of the mathematical model of the direct heat transfer problem for continuous casting. This model serves as a basis for the inverse problem that is discussed in detail in the remainder of the section. The direct problem will also be employed to generate simulated temperature measurements for the application of the proposed inverse analysis algorithms.

The mathematical description of the physical problem consists of

- a convection-diffusion equation for the solid part of the ingot:

$$\nabla^2 T(\mathbf{r}) - \frac{1}{a} v_x \frac{\partial T}{\partial x} = 0 \tag{1}$$

  where $T(\mathbf{r})$ is the temperature at point $\mathbf{r}$, v_x is the casting velocity (assumed to be constant and in the positive x-direction) and a is the thermal diffusivity of the solid phase, and

- boundary conditions defining the heat transfer process along the boundaries ABCDO (Figure 1), including the specification of the melting temperature along the phase change front:

$$T(\mathbf{r}) = T_\mathrm{m}, \quad \mathbf{r} \in \Gamma_{\mathrm{AB}} \tag{2}$$

$$T(\mathbf{r}) = T_\mathrm{s}, \quad \mathbf{r} \in \Gamma_{\mathrm{DO}} \tag{3}$$

$$-\lambda \frac{\partial T}{\partial n} = q(\mathbf{r}) = 0, \quad \mathbf{r} \in \Gamma_{\mathrm{OA}} \tag{4}$$

$$-\lambda \frac{\partial T}{\partial n} = q(\mathbf{r}), \quad \mathbf{r} \in \Gamma_{\mathrm{BC}} \tag{5}$$

$$-\lambda \frac{\partial T}{\partial n} = h[T(\mathbf{r}) - T_\mathrm{a}], \quad \mathbf{r} \in \Gamma_{\mathrm{CD}} \tag{6}$$

where T_m is the melting temperature, T_a is the ambient temperature, T_s is the ingot temperature when leaving the system, λ is the thermal conductivity, h is the convective heat transfer coefficient and q is the heat flux.

In the inverse analysis, the location of the phase change front where the temperature is equal to the melting temperature is unknown. This means that the mathematical description is incomplete and needs to be supplemented by

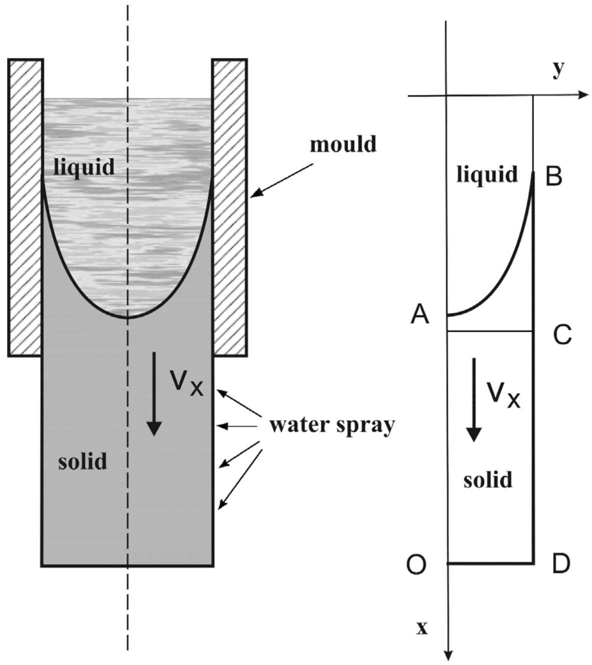

Figure 1.
Schematic of the continuous casting system and the domain under consideration

measurements. Typically, the temperatures U_i are measured at some points inside the ingot (in case of using thermocouples) or on the surface (if an infrared camera is used). These measurements are collected in a vector **U**.

The objective is to estimate components of vector **Y**, which uniquely describes the phase change front location. In this work, two segments of Bezier splines are used to approximate the interface. This means that vector **Y** contains components of the control points defining the Bezier splines.

The ill-conditioned nature of all inverse problems requires that the number of measurement sensors should be appropriate to make the problem overdetermined. This is achieved by using a number of measurement points greater than the number of design variables. Thus, in general, inverse analysis leads to optimization procedures with least squares calculations of the objective functions Δ. However, in the cases studied here, an additional term intended to improve the stability is also introduced (Kurpisz and Nowak, 1995; Nowak, 1997), i.e.

$$\Delta = (\mathbf{T}_{\mathrm{cal}} - \mathbf{U})^{\mathrm{T}}\ \mathbf{W}^{-1}(\mathbf{T}_{\mathrm{cal}} - \mathbf{U}) + (\mathbf{Y} - \tilde{\mathbf{Y}})^{\mathrm{T}}\mathbf{W}_{\mathrm{Y}}^{-1}(\mathbf{Y} - \tilde{\mathbf{Y}}) \rightarrow \min \qquad (7)$$

where vector $\mathbf{T}_{\mathrm{cal}}$ contains temperatures calculated at temperature sensor locations, **U** stands for the vector of temperature measurements and superscript T denotes transpose matrices. The symbol **W** denotes the covariance matrix of measurements. Thus, the contribution of more accurately measured data is stronger than the data obtained with lower accuracy. Known prior estimates of design vector components are collected in vector $\tilde{\mathbf{Y}}$, and $\mathbf{W}_{\mathrm{Y}}$ stands for the covariance matrix of prior estimates. The coefficients of matrix $\mathbf{W}_{\mathrm{Y}}$ have to be large enough to catch the minimum (these coefficients tend to infinity, if prior estimates are not known). It was found that the additional term in the objective function, containing prior estimates, plays a very important role in the inverse analysis, because it considerably improves the stability and accuracy of the inverse procedure.

The present inverse problem is solved by building up a series of direct solutions which gradually approach the correct position of the phase change front. This procedure can be expressed by the following main steps.

- Make the boundary problem well-posed. This means that the mathematical description of the thermal process is completed by assuming arbitrary values $\mathbf{Y}^*$ (as required by the direct problem).
- Solve the direct problem obtained above and calculate temperatures $\mathbf{T}^*$ at the sensor locations.
- Compare the above calculated temperatures $\mathbf{T}^*$ and measured values $\mathbf{U}$, and modify the assumed data $\mathbf{Y}^*$.

Inverse geometry problems are always non-linear. Thus, an iterative procedure is generally necessary. In this procedure, iterative loops are repeated until the newly obtained vector $\mathbf{Y}$ minimizes the objective function (7) within a specified accuracy (Beck and Blackwell, 1988; Kurpisz and Nowak, 1995; Nowak, 1997).

Each iteration loop involves the application of sensitivity analysis (Beck and Blackwell, 1988; Nowak, 1997), which utilizes sensitivity coefficients. According to their definition, these coefficients are the derivatives of the temperature at point i with respect to identified values at point j, i.e.

$$Z_{ij} = \frac{\partial T_i}{\partial Y_j} \tag{8}$$

and provide a measure of each identified value and an indication of how much it should be modified.

Sensitivity coefficients are obtained by solving a set of auxiliary direct problems in succession. Each of these direct problems arises through differentiation of equation (1) and corresponding boundary conditions (2)-(6) with respect to the particular design variable Y_j. Thus, the resulting field Z_j is governed by an equation of the form:

$$\nabla^2 Z_j(\mathbf{r}) - \frac{1}{a} v_x \frac{\partial Z_j}{\partial x} = 0 \tag{9}$$

Differentiation of the boundary conditions (3)-(6) produces conditions of the same type as in the original thermal problem, as follows:

$$Z_j(\mathbf{r}) = 0, \quad \mathbf{r} \in \Gamma_{\mathrm{DO}} \tag{10}$$

$$-\lambda \frac{\partial Z_j}{\partial n} = 0, \quad \mathbf{r} \in \Gamma_{\mathrm{OA}} \tag{11}$$

$$-\lambda \frac{\partial Z_j}{\partial n} = 0, \quad \mathbf{r} \in \Gamma_{\mathrm{BC}} \tag{12}$$

$$-\lambda \frac{\partial Z_j}{\partial n} = hZ_j, \quad \mathbf{r} \in \Gamma_{\mathrm{CD}} \tag{13}$$

The boundary condition along the phase change front Γ_{AB} is also obtained by differentiating equation (2):

$$\frac{\partial T}{\partial Y_j} + \frac{\partial T}{\partial x}\frac{\partial x}{\partial Y_j} + \frac{\partial T}{\partial y}\frac{\partial y}{\partial Y_j} = 0 \tag{14}$$

where the derivatives of x and y with respect to the design variable Y_j depend on the particular geometrical representation of the phase change front (Nowak *et al.*, 2000). In this work, two Bezier splines are used, as discussed in more detail later.

Equation (14) can now be rewritten as

$$Z_j = -\frac{\partial T}{\partial x}\frac{\partial x}{\partial Y_j} - \frac{\partial T}{\partial y}\frac{\partial y}{\partial Y_j} \tag{15}$$

or, taking into account Fourier's law,

$$Z_j = -\frac{1}{\lambda}\left(q_x \frac{\partial x}{\partial Y_j} - q_y \frac{\partial y}{\partial Y_j}\right) \tag{16}$$

where q_x and q_y are the x- and y-components of the heat flux vector.

The Cartesian components of the heat flux vector can be expressed in terms of the tangential and normal components, q_τ and q_n, by the relations:

$$\begin{cases} q_x = -q_n \ \cos(\alpha) - q_\tau \ \cos\left(\frac{\pi}{2} + \alpha\right) \\ q_y = -q_n \ \sin(\alpha) + q_\tau \ \sin\left(\frac{\pi}{2} + \alpha\right) \end{cases} \tag{17}$$

where $\cos(\alpha)$ and $\sin(\alpha)$ are the direction cosines of the normal vector pointing outwards the solid phase (Figure 2).

Taking the above into account, the boundary condition along the phase change front takes the final form:

$$Z_j = -\frac{1}{\lambda}\left\{[-q_n \ \cos(\alpha) + q_\tau \ \sin(\alpha)]\frac{\partial x}{\partial Y_j} + [q_n \ \sin(\alpha) - q_\tau \ \cos(\alpha)]\frac{\partial y}{\partial Y_j}\right\} \tag{18}$$

Solving the above direct problem for the field Z_j, one can collect results at particular measurement points, i.e. $Z_{ij}, \ \ i = 1, 2, \ldots$. Repeating this procedure for all design variables, the whole sensitivity matrix **Z** can then be constructed. This is the most expensive and time consuming stage of the analysis.

Through application of sensitivity analysis and some basic algebraic manipulations (Nowak *et al.*, 2000), minimization of the objective function equation (7) leads to the following set of equations (Nowak, 1997; Nowak *et al.*, 2000):

$$\left(\mathbf{Z}^{\mathrm{T}}\mathbf{W}^{-1}\mathbf{Z} + \mathbf{W}_{\mathbf{Y}}^{-1}\right)\mathbf{Y} = \mathbf{Z}^{\mathrm{T}}\mathbf{W}^{-1}(\mathbf{U} - \mathbf{T}^*) + (\mathbf{Z}^{\mathrm{T}}\mathbf{W}^{-1}\mathbf{Z})\mathbf{Y}^* + \mathbf{W}_{\mathbf{Y}}^{-1}\tilde{\mathrm{Y}} \tag{19}$$

In this work, the BEM is applied for solving both thermal and sensitivity coefficient problems. The main advantage of using this method is the simplification in meshing, as only the boundaries have to be discretized. This is particularly important in inverse geometry problems in which the geometry of the body is changed at each iteration step. Furthermore, the location of the internal measurement sensors does not affect the discretization. Finally, in heat transfer analysis, BEM solutions directly provide temperatures and heat fluxes, both of which are required by inverse solutions. In other words, the numerical differentiation of the temperature field in order to calculate heat fluxes is not needed.

The BEM system of equations for both the thermal and sensitivity coefficient problems has the same form:

$$\mathbf{HT} = \mathbf{GQ} \tag{20}$$

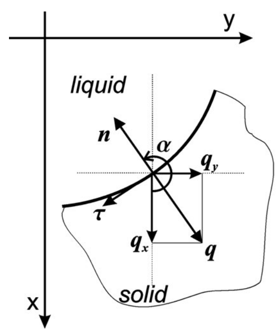

Figure 2.
Geometrical relations on the phase change front

$$\mathbf{H}\mathbf{Z}_j = \mathbf{G}\mathbf{Q}_j^Z \tag{21}$$

where **H** and **G** stand for the BEM influence matrices. The fundamental solution of the two-dimensional convection-diffusion equation is expressed by the following formula, assuming that the velocity field is constant along the x-direction:

$$u^* = \frac{1}{2\pi\lambda}\exp\left(-\frac{v_x r_x}{2a}\right)\ \mathrm{K}_0\left(\frac{|v_x| r}{2a}\right) \tag{22}$$

where $\mathbf{K}_0$ stands for the Bessel function of the second kind and zero order and r is the distance between source and field points, with its component along the x-axis denoted by r_x.

3. Application of Bezier splines

As noted before, the ill-conditioned nature of all inverse problems requires that they have to be made overdetermined. On the other hand, it is very important to limit the number of sensors, mainly because of the difficulties with measurements acquisition. Application of Bezier splines allows the modelling of the phase change front using a much smaller number of design variables.

The Bezier curve (Draus and Mazur, 1991) is built up of cubic segments. Each of these segments is controlled by four control points $\mathbf{V}_0$, $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_3$ (Figure 3). The following formula presents the definition of cubic Bezier segments:

$$\mathbf{P}(u) = (1-u)^3\mathbf{V}_0 + 3(1-u)^2 u\mathbf{V}_1 + 3(1-u)u^2\mathbf{V}_2 + u^3\mathbf{V}_3 \tag{23}$$

where $\mathbf{P}(u)$ stands for a point on the Bezier curve, and u varies in the range $\langle 0, 1\rangle$. This formula has to be differentiated with respect to the design variable Y_j (i.e. the x- and/or y-coordinate of the given control point) in order to obtain derivatives required in the boundary condition (18).

Numerical experiments have shown that a Bezier curve composed of two cubic segments satisfactorily approximates the phase change front. An extra advantage is that the application of Bezier curves permits to limit the number of identified values. In reality, some of these values (coordinates of Bezier control points) are defined by additional constraints resulting from the physical nature of the problem. These conditions are listed below:

- the y-coordinates of the first and the last control points of the Bezier curves ($\mathbf{V}_0^{\mathrm{I}}$, $\mathbf{V}_3^{\mathrm{II}}$ in Figure 4) are known because those points are located on the ingot surface and symmetry axis, respectively;
- the last control point of the first segment, $\mathbf{V}_3^{\mathrm{I}}$, and the first of the second segment, $\mathbf{V}_0^{\mathrm{II}}$, occupy the same position;

- the smoothness of the curve at the connecting points between two Bezier segments is guaranteed if the appropriate control points are collinear (Draus and Mazur, 1991) (compare with Figure 4);
- the equality of the x-coordinate of points $\mathbf{V}_2^{\mathrm{II}}$ and $\mathbf{V}_3^{\mathrm{II}}$ ensures the existence of derivatives on the symmetry axis.

Because of the above conditions only ten quantities have to be estimated, which fully describe the position of the phase change front. Thus, application of the Bezier functions significantly reduces the number of design variables (Nowak *et al.*, 2000), which also means a reduction in the number of required measurements. Acquiring temperature measurements at points located inside the ingot requires to immerse thermocouples in the solidifying material. This perturbs part of the casted material during measurements. The application of an infrared camera is another method of obtaining measurements. Although the first approach seems to be better, because the measurements location can be closer to the identified values, the second does not destroy any casted material and provides measurements which are generally more accurate. Nevertheless, both methods of measuring temperatures always involve measurement errors, which affect the final results.

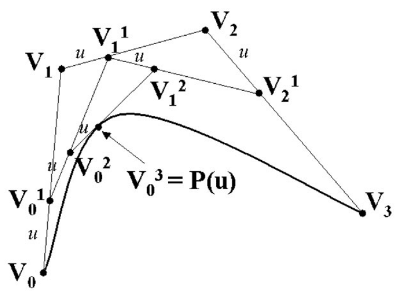

Figure 3.
One Bezier segment and its control points

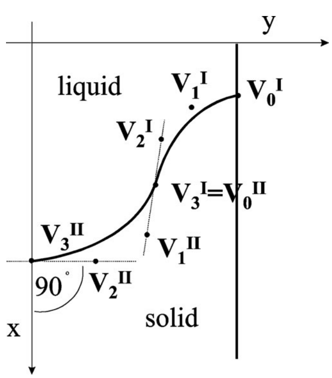

Figure 4.
Identified values in the problem with two Bezier segments

4. Starting point and lumping

Extensive computing of inverse geometry problems showed the great influence of prior estimates and the initial guess on the solution existence and convergence. Contrary to direct problems, the existence of solutions to non-linear inverse problems is not clear. Some starting guesses may not fulfill the conditions for solving the problem. This means that, at the beginning of the iteration process, there is no guarantee that the assumed starting front position (i.e. the starting set of Bezier control points) will lead to the solution.

Because of this, it is proposed (Nowak *et al.*, 2001) that the iteration process is preceded by a kind of *lumping* process. This lumping consists of summing up the coefficients in each row of the main matrix $\mathbf{A} = \mathbf{Z}^{\mathrm{T}}\mathbf{W}^{-1}\mathbf{Z}+\mathbf{W}_{\mathrm{Y}}^{-1}$ of equation (19) and placing the result on the main diagonal of the square matrix $\mathbf{L}$. Thus, matrix $\mathbf{L}$ takes the following form:

$$\mathbf{L} = \begin{bmatrix} \sum_{j=1}^{n} \zeta_{1j} & 0 & \dots & 0 \\ 0 & \sum_{j=1}^{n} \zeta_{2j} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \sum_{j=1}^{n} \zeta_{nj} \end{bmatrix} \tag{24}$$

where ζ_{ij} is an element of the square matrix $\mathbf{A}$. Such matrix decouples the system (19) and each equation may be solved separately.

It was found that replacing matrix $\mathbf{A}$ in equation (19) by $\mathbf{L}$ in the first step of the iteration procedure makes the process always convergent. Simultaneously, in the present inverse geometry problem, application of the lumping procedure turns out to be almost always necessary. An inappropriate initial position of the interface without application of lumping usually leads, very quickly, to results contradicting the physics of the problem. The phase change front in successive iterations appears with very sharp corners, and the iterative process eventually diverges. Such a situation is shown in Figure 5.

Searching for a starting position of the identified values is based on an observation of matrix $\mathbf{L}$. The largest coefficient on the diagonal of matrix $\mathbf{L}$ shows the most sensitive initially-assumed design variables. This initially-assumed coordinate could be the reason for the non-existence of solution, and has to be improved. The direction and value of the correction are determined by solving an appropriate equation from the decoupled system (19). Once this component of vector $\mathbf{Y}^*$ is corrected, the original system (19) with matrix $\mathbf{A}$ can be solved iteratively.

The above algorithm can be further extended in this way, so that not only one component of vector $\mathbf{Y}^*$ is corrected using matrix $\mathbf{L}$, but also all of them. Figure 6 presents a comparison of average errors in subsequent iterations, obtained with the simple and the extended approaches. It can be seen that the final results do not differ significantly. The approach in which all the estimated values are corrected is more time consuming, so the first method seems to be more useful in practical applications.

In the iteration process, it is important that subsequent Bezier control points appear in the correct order. To guarantee the monotonicity of the x- and y-coordinates (without which the Bezier segment makes a loop), the size of the vector $\Delta\mathbf{Y} = \mathbf{Y} - \mathbf{Y}^*$ has to be controlled. If necessary, the calculated vector $\Delta\mathbf{Y}$ may be reduced until the required criterion is fulfilled.

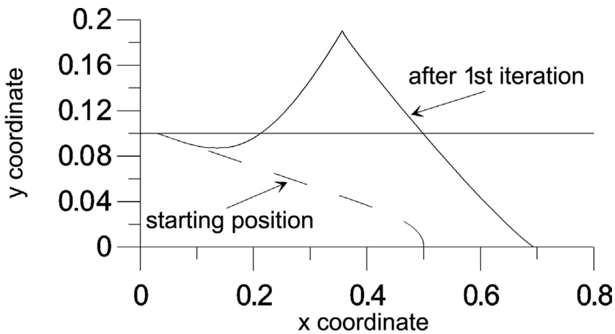

Figure 5.
Estimated curve shape without lumping

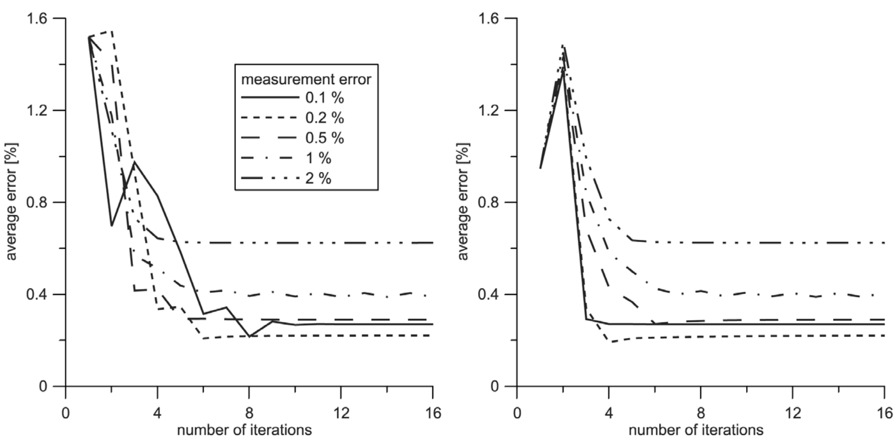

Figure 6.
Comparison of results obtained with correcting one (left) and all (right) estimated values

5. Influence of the number of measurements and their errors on final results

In order to demonstrate the main advantages of the lumping algorithm, a two-dimensional continuous casting problem from the copper industry is solved. The following heat fluxes were adopted in these calculations: $q_{\mathrm{BC}} = 4 \times 10^6\,\mathrm{W/m^2}$ and $q_{\mathrm{CD}} = 4{,}000\ (T - T_{\mathrm{s}})\ \mathrm{W/m^2}$. All the results were obtained for the melting temperature $T_{\mathrm{m}} = 1{,}083°\mathrm{C}$, whereas the end temperature T_{s} was assumed to be 50°C. Temperature measurements were assumed to be read inside the casting material (thermocouples) and along the surface outside the crystallizer (infrared camera).

5.1 Signals recorded with thermocouples

First, the influence of measurement errors on the accuracy of the phase change front location was tested. In general, manufacturers provide information on the maximum temperature errors for measurements carried out by thermocouples, for instance less than 2 per cent. In the analyses carried out here, measurement errors were assumed at five levels, to be less than 0.1, 0.2, 0.5, 1 and 2 per cent. In real conditions, the error variation can be approximated by a normal (Gaussian) distribution. In the present paper, measured temperatures were simulated by adding errors to temperatures obtained from the relevant direct solution. The errors are generated by a random generator with normal and/or uniform distribution.

Figure 7 shows the average temperature errors along the estimated phase change interface, for various levels and distributions of measurement errors, where the estimation of the phase change front location was carried out iteratively. This iterative procedure is terminated when the average temperature error stops changing or its changes do not exceed a given tolerance. In the present work, this average error consists of the difference

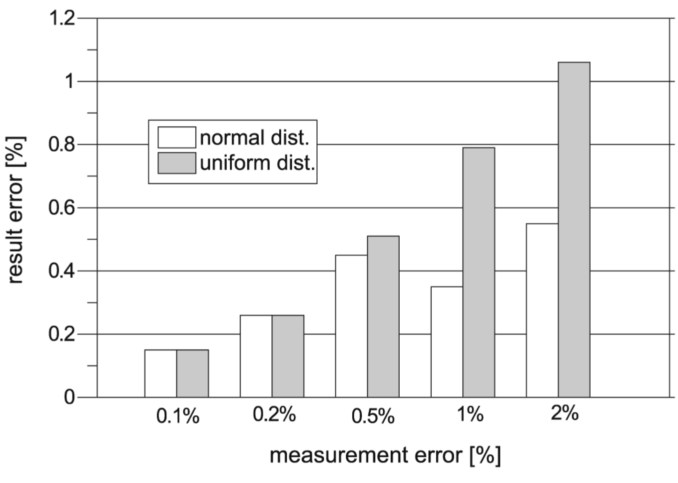

Figure 7.
Average temperature error along the estimated phase change interface with various levels and distributions of error

between the temperature T at a node lying on the Bezier curve (solid-liquid boundary) and the melting temperature T_m, summed over all nodes lying on this interface.

Figure 8 presents the successive locations of the phase change interface and the relevant temperature distribution along this line for normal error distribution and two measurement errors, i.e. 0.5 per cent (case (a)) and 2 per cent (case (b)), respectively.

The influence of the number and location of measurement points was the next issue to investigate. This matter has a significant importance, particularly when the temperature is measured inside the body using thermocouples. In this paper, three different sets of sensors, i.e. sets A, B and C (shown in Figure 9), have been tested. The first and second sets are obtained by immersing five thermocouples in a solidifying material. In set A, the temperature is measured along the estimated boundary, while in set B, sensors are located at the same vertical locations (apart from the bottom one). The last set C consists only of two thermocouples. It can be assumed that each of the thermocouples provide five measurements (at equal time intervals). This means that 25 measurements are obtained for sets A and B, and ten for set C.

For the present problem, the minimum number of measurements necessary to solve the inverse problem is equal to ten. This is because of the application of two Bezier splines to model the phase change front (the number of identified values is equal to ten). Figure 10 shows a comparison of results obtained with

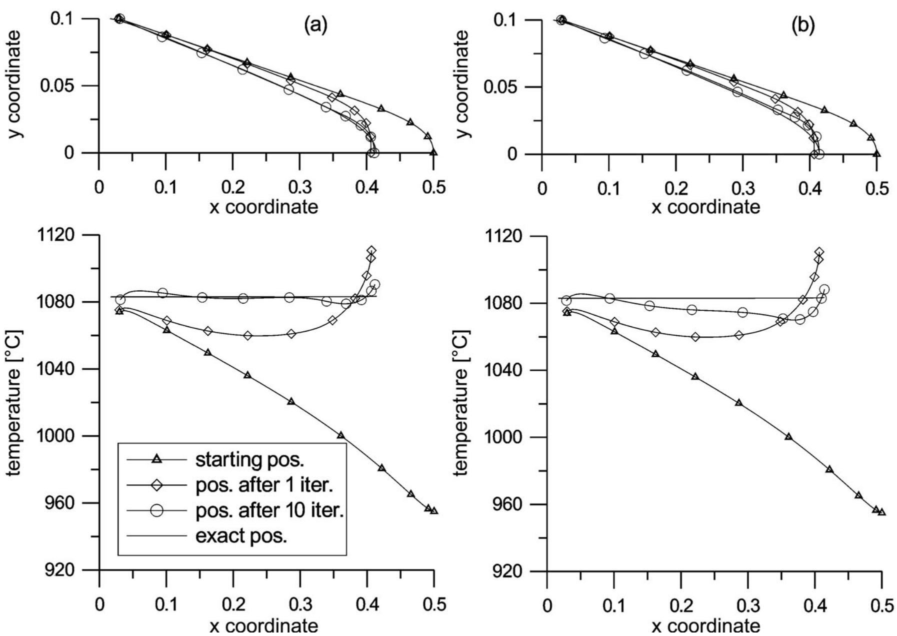

Figure 8.
Location of solid-liquid boundary and temperature distribution along this boundary. (a) mean error 0.5 per cent; (b) mean error 2 per cent

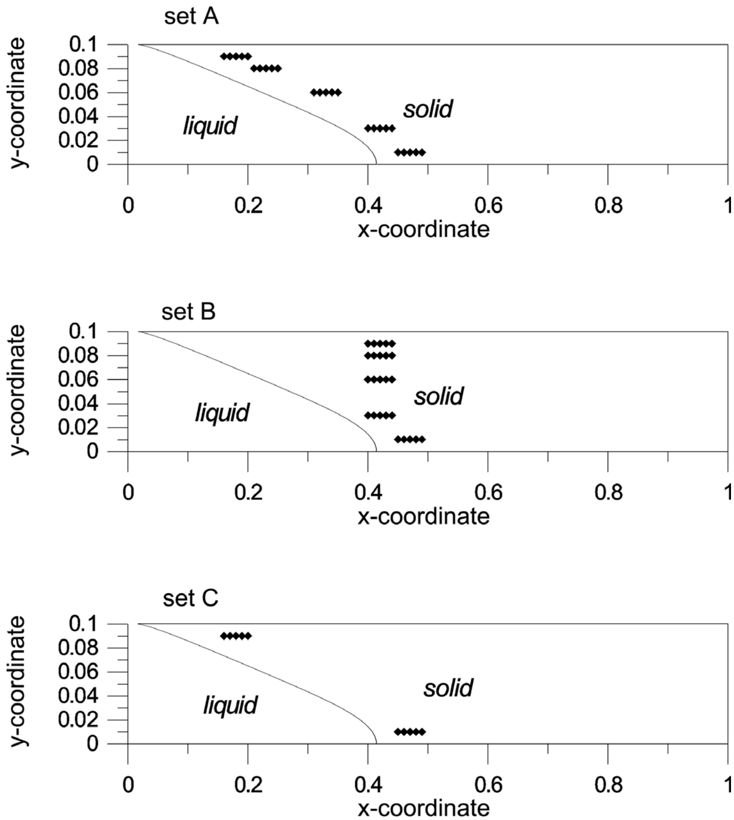

Figure 9.
Three sets of temperature sensors

25 measurements for sets A and B, while similar comparisons for sets A and C with ten measurements are shown in Figure 11. In this case, each thermocouple in set A reads only two temperatures. These figures show that the best results are obtained for small measurement errors and sensors placed close to the identified values.

5.2 Signals recorded with infrared camera

An infrared camera is an alternative and relatively easy way for obtaining temperature measurements. Furthermore, these cameras measure temperatures with small errors, say 0.2 K. Unfortunately, the temperature has to be measured on the surface of the body outside the crystallizer and therefore, the sensor points are located at some distance from the phase change front. On the other hand, there are no strong limitations on the number of measurement points.

Figures 12 and 13 show results obtained by using an infrared camera for solving inverse geometry thermal problems. The first figure shows successive phase change front locations obtained during the iteration process while in

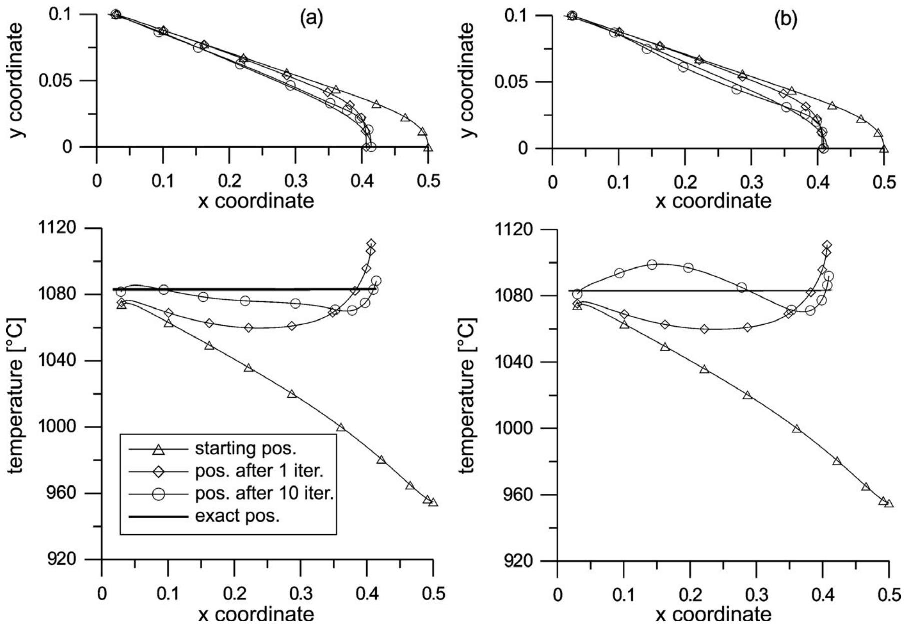

Figure 10.
Comparison of results for sets A and B (25 measurements). (a) mean error 0.5 per cent; (b) mean error 2 per cent

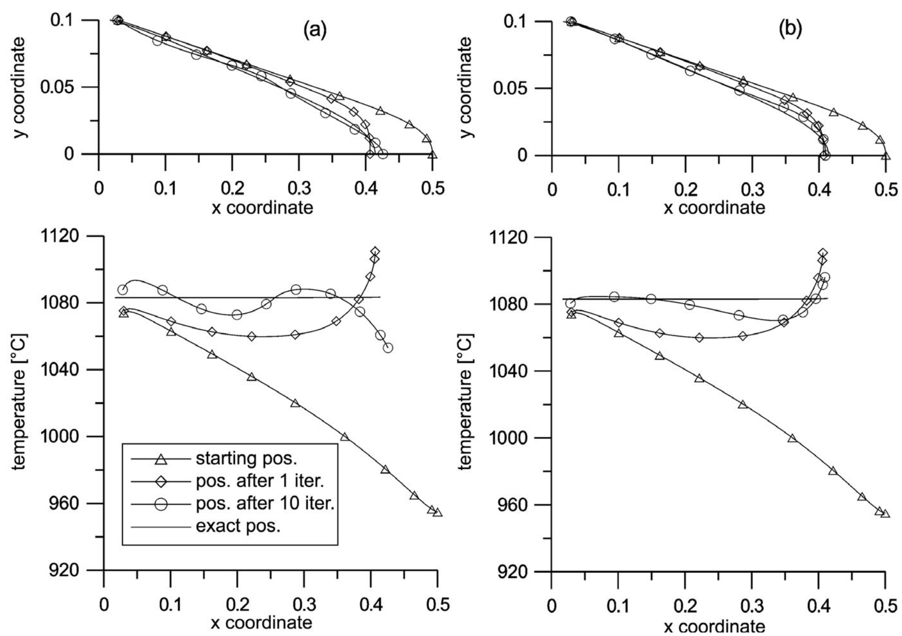

Figure 11.
Comparison of results for sets A and C (ten measurements). (a) mean error 0.5 per cent; (b) mean error 2 per cent

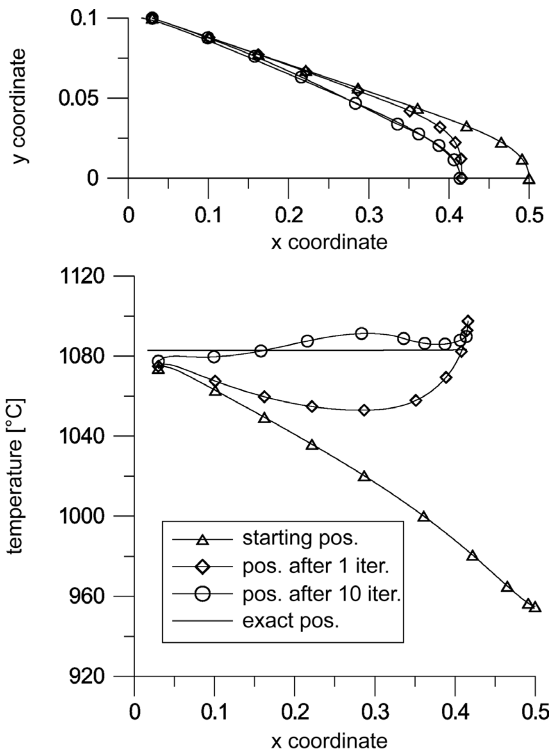

Figure 12.
Front location and temperature along the interface boundary (40 measurements, maximum error 0.2 per cent)

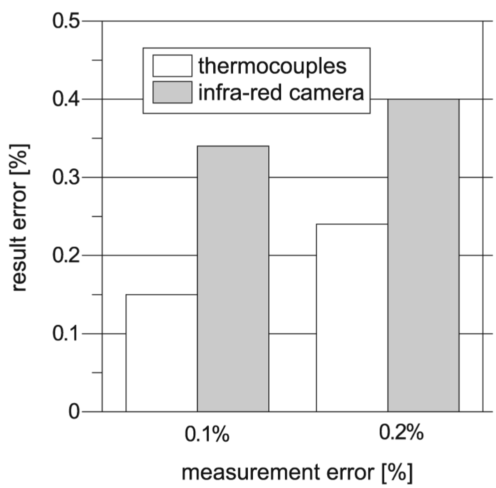

Figure 13.
Comparison of results obtained for thermocouples (25 measurements) and infrared camera (40 measurements)

the second one, the average error is presented. This error consists of the difference between the temperature T at a node lying on the Bezier curve (solid-liquid boundary) and the melting temperature T_{m}, summed over all nodes lying on this front.

A comparison of both methods (i.e. 25 sensors inside the body and 40 measurements obtained from infrared camera) shows that the results obtained for the same measurement errors are better in the case of using thermocouples. On the other hand, it is difficult to obtain measured temperatures with such a low error level. In the case of infrared cameras, the phase change front location is reasonable in view of the costs of the experiment. Furthermore, measurements can easily be repeated as many times as required.

6. Conclusions

This paper presented an algorithm for solving inverse geometry problems in continuous casting. The usefulness of the application of cubic Bezier functions in modelling the phase change boundary has been shown. Using this approach, a significant reduction in the number of identified values and, consequently, the number of measurements have been achieved.

The dependence of the final results on the number, location and accuracy of measurements was investigated. Temperatures were assumed to be measured using thermocouples and/or infrared cameras. The results obtained with both methods were presented and compared.

Some modifications to the solution algorithm, providing faster convergence of the iteration process, have also been discussed. These modifications consist of guessing the initial phase change front position employing a lumping procedure. The paper also demonstrated the applicability of sensitivity analysis to phase change heat transfer processes.

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